ASSORTMENT OPTIMIZATION

- Although we want choice models that capture a wide range of customer behavior, we also want the models to be easy to optimize over.
- The main pricing problem related to choice models is assortment optimization: given a set of items (with given prices) and a choice model, we want to offer a subset of items to maximize revenue
- Formally: Given item set $N=\{1,2,\dots,n\}$, choice model π s.t $\pi_j(S) = \mathbb{P}[\text{Item } j \text{ purchased from set } S\subseteq N]$.
  - Item j has price p_j
  - If set S is offered, revenue $R(S) = \sum_{j\in S} \pi_j(S)\, p_j$
  - Aim - pick $\boxed{S^* = \underset{S\subseteq N}{\arg\max}\, R(S)}$

- Note - # of subsets of $N = 2^n - 1$

  In general, finding S^* may be difficult

  - However, for the MNL model, it can be found efficiently using a simple algorithm!

Assortment opt under the MNL model

- Recall - For the MNL, $\exists\ v_j > 0$ for each item j s.t.

$$\pi_j(S) = \frac{v_j}{v_0 + \sum_{i \in S} v_i} = \frac{v_j}{1 + \sum_{i \in S} v_i} \quad v(S)$$

can assume $v_0 = 1$ by normalizing

$$\Rightarrow \quad R(S) = \sum_{j \in S} \frac{v_j p_j}{1 + v(S)}, \quad R^* \triangleq \max_{S \subseteq N} R(S)$$

- Thm (Gallego, Tallurid Van Ryzin) · Let $p_1 \geq p_2 \geq \dots \geq p_n$, and

  $E_0 = \phi,\ E_1 = \{1\},\ E_2 = \{1, 2\},\ \dots,\ E_n = N$

  (nested-by-revenue sets)

  Then $\exists\ k^* \in \{0, 1, \dots, n\}$ s.t. $E_{k^*} \in \underset{S \subseteq N}{\operatorname{argmax}}\, R(S)$

  (i.e., there is some nested-by-revenue set which has opt revenue!)

Pf - By definition, we know $\forall S \subseteq N$

$$R^* \geq \frac{\sum_{j \in S} v_j P_j}{1 + \sum_{i \in S} v_i}$$

$$\Rightarrow \boxed{R^* \underset{\text{equal for } S^*}{\geq} \sum_{j \in S} v_j (P_j - R^*)} \quad \forall S \subseteq N$$

- Now suppose an oracle told us R^*; then in order to find S^*, we can instead find:

$$(S^* =)\ S' \equiv \arg\max_{S \subseteq N} \sum_{j \in S} v_j \underbrace{(P_j - R^*)}_{\text{'virtual price'}}$$

- The solution to the above problem is to pick $S' = \{ j \mid P_j \geq R^* \}$
- Observe that $S' = E_k$ (nested-by-revenue set) for some k
- What if we do not know R^*?

We can still search over the $\{E_k\}$ to find the best!

$$\Rightarrow \quad S^* = S' = \max_{k \in \{0,1,\dots,n\}} [R(E_k)]$$

Eg - (Assortment Opt under MNL)

5 items, prices $(P_1, P_2, \ldots, P_5) = (7, 6, 4, 3, 2)$

MNL parameters $(v_1, v_2, \ldots, v_5) = (3, 5, 6, 4, 5)$, $v_0 = 10$

Assortment S	{1}	{1,2}	{1,2,3}	{1,2,3,4}	{1,2,..,5}
R(S)	1.615	2.833	3.125	3.107	2.939

However, ~~things~~ these problems are much harder to solve for general $\Pi_j(S)$!

Eg - (2-class MNL)

2 classes {a, b}, $\alpha^a = 0.5$, $v^a = (5, 20, 1)$, $v_0^a = 1$

$\alpha^b = 0.5$ $v^b = (1/5, 10, 10)$, $v_0^a = 1$

Prices $\equiv (8, 4, 3)$

Then: Opt for class a $\equiv \{1\}$, $R^* = 20/3$

Opt for class b $\equiv \{1,2\}$, $R^* = 26/7$

Opt for mixture $\equiv \underbrace{\{1,3\}}_{\text{not nested-by-revenue!}}$, $R^* = 4.48$

Thm - 2-class MNL assortment optimization is NP-complete.