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Problem 1: Understanding the Network RM Value Function

An airline operates a flight network among 3 locations A, B and C, as shown in the figure below.

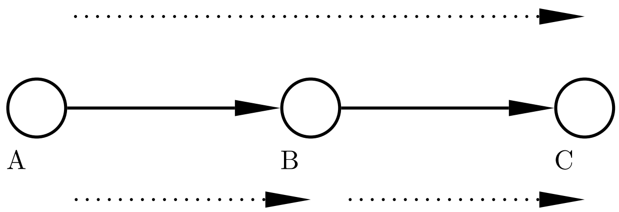

Figure 1: Two-leg airline network, with flights $A \to B$ and $B \to C$. We want to sell itineraries (A, B), (B, C) and also (A, C); the latter requires reserving one seat on each leg.

Assume that we can have at most one itinerary request at each time period. A part of the value function for time period $t + 1$ has been computed and given in the following table.

Remaining Capacity for B-C→ Remaining Capacity for A-B ↓	0	1	2	3
0	0	600	900	1000
1	300	700	1000	1300
2	400	800	1100	1300
3	500	850	1100	1300

For example, if the remaining capacity on flight leg A-B is 2 and the remaining capacity on flight leg B-C is 1, then the value of the value function at time period $t + 1$ is $V_{t+1}([2, 1]) = 800$.

Part (a)

Assume that the remaining capacity on flight leg A-B is 1 and the remaining capacity on flight leg B-C is 2 at time period t. Suppose we get a request for a $A - B$ flight for \$150. Is it optimal to accept a this request? Why?

Part (b)

Assume that the remaining capacity on flight leg A-B is 2 and the remaining capacity on flight leg B-C is 3 at time period t. Is it optimal to accept a request for a $A - C$ itinerary for \$275 at time period t? Why?

Problem 2: LP-based Allocations and Bid-prices for Network RM

In this problem, we will implement the LP-based approximation methods we studied in class to derive allocation decisions for a simple network.

We want to sell tickets on two flights, Flight 1 from San Francisco (SFO) to Detroit (DIA), and Flight 2 from DIA to St. Louis (STL); a person flying from SFO to STL needs a seat on both flights. Consider selling tickets over 300 time periods (for example, think of 60 days, with 5 time periods per day), with initial capacity of 100 on Flight 1 (SFO-DIA), and 120 on Flight 2 (DIA-STL). The airlines offers a total of 6 products. In particular, for each origin-destination (OD) pair, there are two fare classes – class Y corresponding to the full-fare economy class ticket, and class M corresponding to the discount-fare economy class ticket. In each period, the airlines gets at most one request for a particular product, where product j is requested with probability λ_j. The fares and per-period request probabilities for the products are given in the following table. Using

Product	Class	Total Fare	Per-period Arrival Probability
SFO-DIA	Y	\$150	1/10
SFO-DIA	M	\$100	1/5
DIA-STL	Y	\$120	1/15
DIA-STL	M	\$80	4/15
SFO-STL	Y	\$250	1/10
SFO-STL	M	\$170	2/15

Part (a)

Let D_0 be the number of time-periods in which no request arrived, and D_j be the (random) demand for product j. What is the distribution, and average value of D_0? What about for D_1?

Part (b)

Let $V_t(\underline{\mathbf{x}})$ be the value function at beginning of period t with state $\underline{\mathbf{x}}$. Write down a linear program that computes the *randomized-LP upper bound* $V_{300}^{UB}(\underline{\mathbf{x}})$, as well as the *fluid upper bound* $V_{300}^{FL}([100, 120])$. How do $V_{300}(\underline{\mathbf{x}})$, $V_{300}^{UB}(\underline{\mathbf{x}})$ and $V_{300}^{FL}(\underline{\mathbf{x}})$ relate to each other?

Part (c)

Solve the fluid upper-bound LP to compute approximate seat allocations for each class, and the fluid upper bound $V_{300}^{FL}([100, 120])$
Hint: You should write a function to solve the LP given the RHS values of the constraints; you will need to use this in the next few parts

Part (d)

Assume that the $\{D_j\}$, the *total* demand (i.e., total number of requests over time) for each product j is an independent random variables with the distribution as in part *a*. Find the randomized-LP upper bound $V_{300}^{UB}([100, 120])$ via simulation in this case, and compare with the fluid bound.

Part (e)

Write down the dual of the fluid upper-bound LP, and use it to compute bid prices for each flight.

Part (f)

Next, generate sample arrival sequences, and find the average revenue earned if we admit product requests according to (i) admit all feasible incoming request, (ii) booking limits for each product class as computed via the fluid LP in part (d), and (iii) the bid-price policy (i.e., where we admit products whose price is greater than the sum of bid-prices of its required resources) from part (e).

Part (g)

Next, suppose the system becomes twice as busy; we now have 10 periods per day, with at most one arrival every period (and thus we have 600 time periods in 60 days) with the same arrival probabilities as before. In response, the airline announces two flights from SFO to DIA (total capacity 200), and two flights from DIA to STL (total capacity 240). Argue that the bid-prices you compute in part (e) *remain the same* in this setting. What is the new fluid upper-bound?

Part (h)

We now want to see how our bid-price policy does when the system becomes bigger. Let $R^{(k)}$ denote the average revenue, and $V^{FL,(k)}$ denote the fluid upper bound, in the system where all capacities and all demands are scaled by k (for example, in part g, we had $k = 2$). Repeat the simulation from part (f) with capacities and demands increased by a factor of $k = 2, 4, 6, 8, 10$, and in each case, plot $\frac{R^{(k)} - V^{FL,(k)}}{V^{FL,(k)}}$.

Part (i)

(Optional) Repeat part (f), but now recompute the bid-prices at the start of each day.