Sid Banerjee (sbanerjee@cornell.edu)

Problem 1: Demand learning and the spiral-down effect

In this problem, you will simulate joint learning and setting of protection-levels, in settings with perfect segmentation (where simple demand estimation gives the optimal protection level), and with a more general customer-choice model (where you get the spiral-down effect).

Part (a)

For all parts in this problem, we will consider making reservations for a flight with $c = 100$ seats, using two fare classes with prices $p_1 = \$200$ and $p_2 = \$100$. Write a simulator which generates a stream of T arrivals, and for any given protection level x, returns the number of tickets sold of fare class 1 and fare class 2, as well as the total revenue.

To test your simulator, assume that $T = 200$, each incoming request is of type $\{0, 1, 2\}$ (where 0 means no request, 1 means request for fare-class 1, and 2 means request for fare-class 2) with probabilities $\lambda = \{0.1, 0.3, 0.6\}$, and the protection levels are 0 and 100.

Note: Remember that the protection level indicates the minimum number of seats you should preserve for high-fare customers. In particular, when you sell a high-fare ticket, you need to reduce both the remaining capacity and the protection level.

Part (b)

Next, for the same system parameters ($c = 100, p = \{200, 100\}, \lambda = \{0.1, 0.3, 0.6\}$), and for number of arrivals $T = 100$ and $T = 200$, generate 1000 sample-paths of request arrivals, and for each path, find the revenue under each possible protection level $x \in \{0, 1, 2, \ldots, 100\}$. Plot the expected revenue vs. protection-level, and verify that the maximum revenue corresponds to the optimal protection-level (from Littlewood's rule)

Hint: Since $p_1/p_2 = 1/2$, the optimal protection-level x^* corresponds to the median of F_1, i.e., it satisfies $\mathbb{P}(D_1 \leq x^*) \geq 1/2$. What is the distribution of D_1?

Part (c)

Now, we want to set the protection-level based on past sales-data. Again set system parameters as $c = 100, p = \{200, 100\}, \lambda = \{0.1, 0.3, 0.6\}$, and for number of requests $T = 100$, simulate 1000 flights. For the first 50 flights, set the protection-level to 100; for each subsequent flight, compute the optimal protection-level using the empirical distribution of fare-class 1 tickets sold in previous flights. Plot the protection-level vs. number of flights for both $T = 100$ and $T = 200$.

Part (d)

For $T = 100$ and 200, compare the estimated protection level after 1000 flights with the optimal level computed in part (b); are they the same? Also, plot the empirical distribution of fare-class 1 tickets sold, and compare to the true distribution. Can you explain any discrepancies in the distributions in the two cases?

Part (e)

As in the discussion on the spiral-down effect, we now assume that we have a third type of agents who are want a fare-class 2 ticket, but if that is unavailable, are willing to buy a fare-class 1 ticket. Formally, assume that each incoming request is of type $\{0, 1, 2, 3\}$ (where 3 represents the new price-conscious type) with request-probabilities $\lambda = \{0.1, 0, 0.4, 0.5\}$ (i.e, there are no agents who only want fare-class 1 tickets, but half the requests are from agents who are willing to buy fare-class 1 if fare-class 2 is unavailable).

For $T = 100$, and for 1000 flights, repeat the procedure in part (c), setting the protection-level for the first 50 flights to c, and for each subsequent flight via Littlewood's rule on the empirical distribution of fare-class 1 tickets sold in previous flights.

Part (f)

Finally, in the above setting (with $T = 100$ and request-probabilities $\lambda = \{0.1, 0, 0.4, 0.5\}$), find the optimal protection-level as in part (b) (by finding the expected revenue over 1000 samples for each protection-level in $\{0, 1, \ldots, 100\}$).