ORIE 4154 - Pricing and Market Design

Module 1: Capacity-based Revenue Management
(Two-stage capacity allocation, and Littlewood's rule)

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Cornell University

RM in the Airline Industry

Here's the Story…

- Until 1978, US airline industry was heavily regulated
  - US Airline Deregulation Act can in 1978
  - Price controls lifted
  - Free entry and exit from markets
- This led to the rise of new low-cost carriers
  - They provide bare-bone service, passengers paying for meals and all luggage handling, non-union employees
  - Their service structure allow them to offer low fares
  - One such carrier was People Express, started in 1981

Courtesy: Huseyin Topaloglu

RM in the Airline Industry

What Happened to Major Airlines

- Major airlines were heavily affected, especially by the loss of discretionary leisure travelers
- Dilemma faced by American Airlines
  - If it matched People Express' fares, it can retain customers but not cover cost
  - If it does not, then it would lose customers

Courtesy: Huseyin Topaloglu

RM in the Airline Industry

American Airlines' Solution

- Bob Crandall, VP of Marketing at AA then, recognized the following key facts
  - Many AA flights departed with empty seats
  - Marginal cost of using these seats was very small
  - AA could use these "surplus seats" to compete on cost

Courtesy: Huseyin Topaloglu

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But how?

- Create new restricted, discounted fares called "Super Saver" and "Ultimate Super Saver" fares
  - Must book at least 2 weeks prior to departure and stay at destination over a Saturday night
  - Passengers not meeting this restriction are charged a higher fare
  - Restrict number of discount seats sold on each flight to save seats for full-fare passengers book late
  - DINAMO – Dynamic Inventory Allocation and Maintenance Optimizer
- People Express allowed every seat to be sold at a low fare!

Courtesy: Huseyin Topaloglu

RM in the Airline Industry

Results of the New Strategy

- AMR shares initially plunged on announcement of "Ultimate Super Saver" fares Jan. 1985
  - Analysts thought it was the start of a price war
  - "American cannot operate profitably at these fares"
- DINAMO proved to be surprisingly effective
  - AA total revenues rose
  - Competitors suffered: e.g. People Express

    1984 $60M profit (all-time high) ← DINAMO
    1985 $160M loss
    1986 Bankruptcy, sold to Continental

Courtesy: Huseyin Topaloglu

Problem: Single-resource two-stage capacity allocation

Want to maximize revenue from selling multiple copies of a single resource (e.g., C seats on a single flight)

Problem: Single-resource two-stage capacity allocation

- Buyer behavior:
  - (Dynamics) Buyers arrive sequentially to the market
  - (Choice) Each buyer wants either a discount-fare (i.e., low price) ticket or a full-fare (i.e., high-price) ticket

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  - (Choice) Each buyer wants either a discount-fare (i.e., low price) ticket or a full-fare (i.e., high-price) ticket
- Seller constraints:
  - (Capacity) Has C identical units (seats) to sell
  - (Prices) Prices fixed to p_h (full-fare) and p_l (discount fare), with $p_h > p_l$
  - (Control) Can choose how many discount-fare and full-fare tickets to sell

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  - (Control) Can choose how many discount-fare and full-fare tickets to sell
- Information structure:
  - (Dynamics) All discount-fare customers arrive before full fare customers
  - (Demand Distributions) Demand for full-fare tickets is $D_h \sim F_h$, discount fare tickets is $D_l \sim F_j$

Aside: Customer Segmentation (Price Discrimination)

In our first lecture, we considered a single customer with (unknown) value V for an item, and we charge a single price p

- Note: The best achievable revenue is V
- Customer segmentation: use pricing to get revenue closer to V

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Third-degree (Direct segmentation): use some 'feature' to segment buyers into classes, and charge different price to each class (e.g., student discounts)

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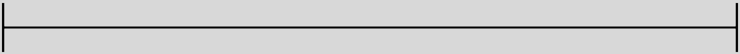
Profits and information requirements increase going up the list

Single-resource two-stage capacity allocation

Timeline of optimization problem



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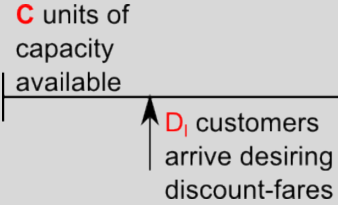

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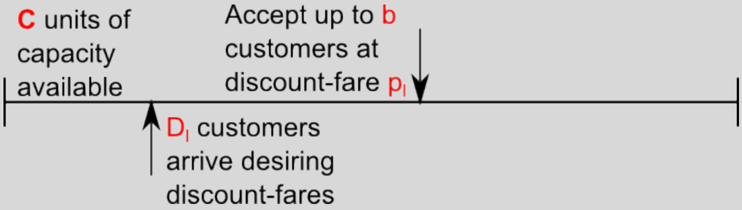

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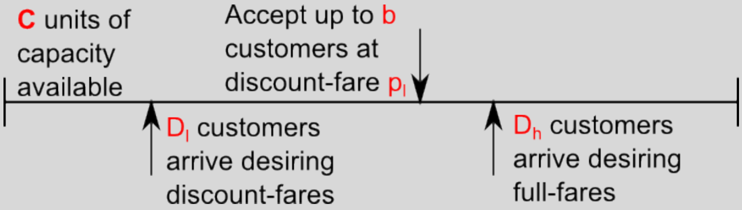

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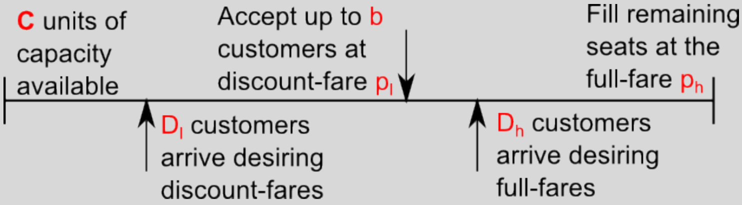

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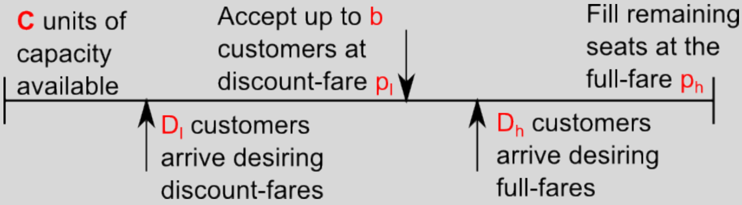

Inputs: prices p_ℓ, p_h, demand distributions F_ℓ, F_h

Control variable: Booking limit b for discount-fare seats

Revenue (as a function of b, D_ℓ and D_h):

$$R(b, D_\ell, D_h) = ?$$

Single-resource two-stage capacity allocation

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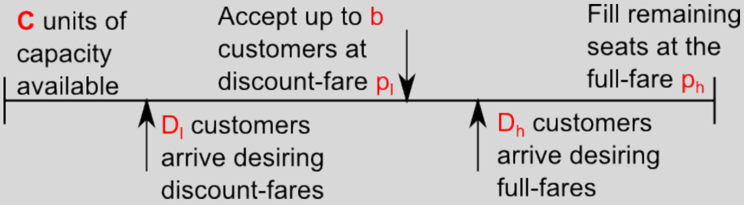

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$$R(b, D_\ell, D_h) = p_\ell \min\{b, D_\ell\} + p_h \min\{D_h, \max\{C - b, C - D_\ell\}\}$$

Single-resource two-stage capacity allocation

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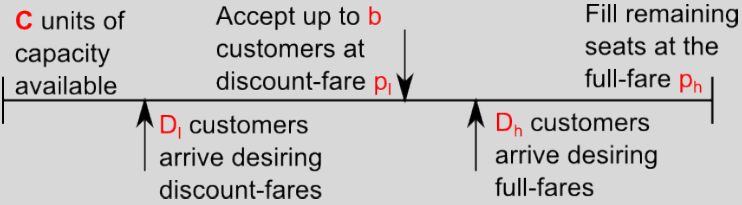

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Aim: Choose $b^* = \arg\max_{b\in[0,C]} \mathbb{E}[R(b, D_\ell, D_h)]$

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Heuristic derivation of Littlewood's rule

The problem

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Marginal-revenue heuristic: Suppose we have y units left, and a discount-fare customer arrives

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- If we accept, then $\Delta R = p_\ell$
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Thus, optimal protection level $y^* = \max_{y \in \mathbb{N}} \left\{ \mathbb{P}[D_h \geq y] > \frac{p_\ell}{p_h} \right\}$

Single-resource two-stage capacity allocation

Formal derivation 1: Continuous RV

$$R^* = \max_{b\in[0,C]} \mathbb{E}\left[p_\ell \min\{b, D_\ell\} + p_h \min\{D_h, \max\{C-b, C-D_\ell\}\}\right]$$

- Assume D_h, D_ℓ are continuous, b can be fractional

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(By linearity of expectation)

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$$= p_\ell \cdot \left(\int_{-\infty}^{b} x \cdot f_\ell(x)dx + \int_{b}^{\infty} b \cdot f_\ell(x)dx\right) + \ldots$$

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$$\left.\int_{b}^{\infty} \mathbb{E}[\min\{C-b, D_h\}] \cdot f_\ell(x)dx\right)$$

Single-resource two-stage capacity allocation

Formal derivation 1: Continuous RV

Thus we want to choose b to maximize:

$$r(b) = \mathbb{E}[R(b, D_\ell, D_h)] = p_\ell \cdot (L_1(b) + L_2(b)) + p_h \cdot (H_1(b) + H_2(b))$$

Where

$$L_1(b) = \int_{-\infty}^{b} x \cdot f_\ell(x) dx$$

$$L_2(b) = \int_{b}^{\infty} b \cdot f_\ell(x) dx$$

$$H_1(b) = \int_{-\infty}^{b} \mathbb{E}[\min\{C - x, D_h\}] \cdot f_\ell(x) dx$$

$$H_2(b) = \int_{b}^{\infty} \mathbb{E}[\min\{C - b, D_h\}] \cdot f_\ell(x) dx$$

We now need to check the first-order condition $\frac{dr(b)}{db} = 0$

Aside: Leibniz rule of integration

Let $f(x,t)$ be such the partial derivative w.r.t. t exists and is continuous. Then:

$$\frac{d}{dx}\left[\int_{A(x)}^{B(x)} f(x,t)dt\right] = \int_{A(x)}^{B(x)} \frac{\partial f(x,t)}{\partial x}dt + \ldots$$
$$f(x,B(x))\frac{dB(x)}{dx} - f(x,A(x))\frac{dA(x)}{dx}$$

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As an example, consider $L_2(b) = \int_b^\infty b \cdot f_\ell(x)dx$:

$$\frac{dL_2(b)}{db} = \int_b^\infty \frac{\partial b f_\ell(x)}{\partial b}dx - b f_\ell(x)\Big|_{x=b} \cdot \frac{db}{db} = \int_b^\infty f_\ell(x)dx - b f_\ell(b)$$

Single-resource two-stage capacity allocation

Formal derivation 1: Continuous RV

$$\frac{dr(b)}{db} = p_\ell \cdot \left(\frac{dL_1(b)}{db} + \frac{dL_2(b)}{db} \right) + p_h \cdot \left(\frac{dH_1(b)}{db} + \frac{dH_2(b)}{db} \right)$$

where we have

$L_1(b) = \int_{-\infty}^{b} x \cdot f_\ell(x)dx$, $L_2(b) = \int_{b}^{\infty} b \cdot f_\ell(x)dx$

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- Differentiating we have (check these for yourself):

$$\frac{dL_1(b)}{db} = bf_\ell(b) \ , \ \frac{dL_2(b)}{db} = \mathbb{P}[D_\ell \geq b] - bf_\ell(b)$$

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$$\frac{dH_1(b)}{db} = \mathbb{E}[\min\{C-b, D_h\}]f_\ell(b)$$

$$\frac{dH_2(b)}{db} = -\mathbb{E}[\min\{C-b, D_h\}]f_\ell(b) + \frac{d\mathbb{E}[\min\{C-b, D_h\}]}{db}\int_b^\infty f_\ell(x)dx$$

Single-resource two-stage capacity allocation

Formal derivation 1: Continuous RV

- Combining all terms, we have

$$\frac{dr(b)}{db} = p_\ell \mathbb{P}[D_\ell \geq b] + p_h \left(\frac{d\mathbb{E}[\min\{C-b, D_h\}]}{db} \right) \mathbb{P}[D_\ell \geq b]$$

Setting $\frac{dr(b)}{db} = 0$, we get $\frac{d\mathbb{E}[\min\{C-b, D_h\}]}{db} + \frac{p_\ell}{p_h} = 0$.

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- Finally, we can again use the Leibniz rule to simplify the LHS

$$\frac{d\mathbb{E}[\min\{C-b, D_h\}]}{db} = \frac{d}{db} \left(\int_{-\infty}^{C-b} x f_h(x) dx + \int_{C-b}^{\infty} (C-b) f_h(x) dx \right)$$

$$= -(C-b) f_h(C-b) + (C-b) f_h(C-b) - \mathbb{P}[D_h \geq C-b]$$

Single-resource two-stage capacity allocation

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$$= -(C-b) f_h(C-b) + (C-b) f_h(C-b) - \mathbb{P}[D_h \geq C-b]$$

Thus, the optimal b^* satisfies: $\mathbb{P}[D_h \geq c - b^*] = \frac{p_\ell}{p_h}$, and hence:

$$C - b^* = y^* = F_h^{-1}\left(1 - \frac{p_\ell}{p_h}\right)$$