ORIE 6180: Design of Online Marketplaces **Spring 2016**

Lecture 10 — March 11

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We used Leibniz integral rule in this lecture

10.1 Overview of the last lecture

In last lecture, we talked about Linkage Principle: the more closely the winning bidder's payment is linked to his signal, the higher is the expected revenue.

10.2 Overview of this lecture

In this lecture, we will introduce a new topic: price theory of 2-sided platforms.

10.3 Price theory of 2-sided platforms

10.3.1 Examples of 2-sided platforms

Here are some real-life examples of 2-sided platforms:

1. Newspaper: Reader v.s. Advertiser. Newspapers get paid by advertisers by putting up advertisements, but may lose readers if they have too many advertisements.

2. Credit Card Companies: Customers v.s. Merchants. Customers generally prefer credit cards and are likely to purchase more when using credit cards, but credit card companies charge a per transaction fee from merchants and merchants might increase the price for credit card customers or set lower bounds for credit card purchases.

10.3.2 The basic model of 2-sided platforms

As observed in [1], the outcomes of a transaction in a 2-sided marketplace depend not only on value or cost but also on the **payment structure**.
The model proposed by Weyl in [2] is structured as follows:

- Two Sides: L (left) and R (right).
- Number of people on both sides are normalized to unit mass.

- Users of side S derives 2 benefits from the platform. (Both values are scaled and can be negative.)
  - · Membership benefit B^S,
  - · Interaction benefit b^S

In this model, the utility functions are:

$$U^L = B^L + b^L N^R - P^L(N^R) \tag{10.1}$$

$$U^R = B^R + b^R N^L - P^R(N^L) \tag{10.2}$$

Note that under these assumptions, the user values are exogenous and quasi-linear. We also assume that there are only homogeneous cross-network interactions and that there are no price discrimination within a side.

10.3.3 Analysis of the model

Assume that the distribution of the benefits are $F^L(B^L, b^L)$ and $F^R(B^R, b^R)$, which lie on some convex set in $\mathbb{R}^2$ and have density functions f^L and f^R.
By the IR constraint, users join the platform if and only if $U^S \geq 0$, which means

$$B^L \geq P^L(N^R) - b^L N^R \quad \text{and} \quad B^R \geq P^R(N^L) - b^R N^L$$

. Then,

$$N^L(N^R, P^L) = \int_{-\infty}^{\infty} \int_{P^L(N^R) - b^L N^R}^{\infty} f^L(b^L, B^L) dB^L db^L \tag{10.3}$$

,

$$N^R(N^L, P^R) = \int_{-\infty}^{\infty} \int_{P^R(N^L) - b^R N^L}^{\infty} f^R(b^R, B^R) dB^R db^R \tag{10.4}$$

.

As shown in Figure 10.1, the shaded area represents N^L. We call users on the dotted line 'marginal users' since they have 0 utility and call users in the shaded area 'loyal users'.

The system can have multiple equilibriums. For example, let $b^L = b^R = 1$, $B^L = B^R = 0$ and $P^L = P^R = \frac{1}{2}$. Then the system reaches equilibrium when $(N^L, N^R) = (0, 0)$ or $(1, 1)$.

Now we assume that the platform wants to maximize welfare or revenue.
Observe that whenever we increase P^L, the dotted line in Figure 10.1 shifts up and N^L decreases. Hence,

$$\frac{\partial}{\partial P^L} N^L(N^R, P^L) < 0$$

. Then, $\tilde{P}^L(N^R \mid N^L)$ is well defined as the inverse of $N^L(N^R, P^L)$. Similarly, we have $\tilde{P}^R(N^L \mid N^R)$.

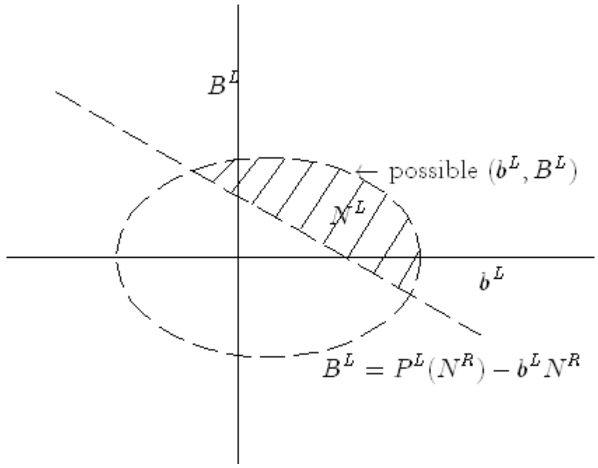

Figure 10.1. Loyal and Marginal Users

Suppose we want to achieve $(N^L, N^R) = (\hat{N}^L, \hat{N}^R)$, then we may define the **insulating tariffs** as

$$P^L(N^R) = \tilde{P}^L(N^R \mid \hat{N}^L)$$

and

$$P^R(N^L) = \tilde{P}^R(N^L \mid \hat{N}^R)$$

.
The value to side L is

$$V^L(N^L, N^R) = \int_{-\infty}^{\infty} \int_{P^L - b^L N^R}^{\infty} (b^L N^R + B^L) f^L(b^L, B^L) dB^L db^L \tag{10.5}$$

. Similarly, we can define $V^R(N^L, N^R)$.
The social welfare is

$$W(N^L, N^R) = V^L(N^L, N^R) + V^R(N^L, N^R) - c^L N^L - c^R N^R - cN^L N^R. \tag{10.6}$$

where c^L, c^R and c are platform costs.
We want to choose (N^L, N^R) to maximize $W(N^L, N^R)$.

$$\frac{\partial W}{\partial N^L} = \frac{\partial V^L}{\partial N^L} + \frac{\partial V^R}{\partial N^L} - c^L - cN^R$$

.
Using Leibniz integral rule and Tonelli's theorem, we can differentiate (10.3) with respect to N^L,

$$1 = (-\int_{-\infty}^{\infty} f^L(b^L, P^L - b^L N^R) db^L) \cdot (\frac{\partial P_L}{\partial N^L})$$

. Then,

$$\frac{\partial P^L}{\partial N^L} = -\frac{1}{\int_{-\infty}^{\infty} f^L(b^L, P^L - b^L N^R) db^L}$$

Differentiate (10.3) with respect to N^R, we get

$$0 = \int_{-\infty}^{\infty} f^L(b^L, P^L - b^L N^R)(\frac{\partial P_L}{\partial N^R} - b^L) db^L$$

. Thus,

$$\frac{\partial P_L}{\partial N^R} = \frac{\int_{-\infty}^{\infty} b^L f^L(b^L, P^L - b^L N^R) db^L}{\int_{-\infty}^{\infty} f^L(b^L, P^L - b^L N^R) db^L}$$

.

Let $\tilde{b}^R(N^L, N^R)$ represent the average interaction value of marginal users. Then,

$$\frac{\partial P^L}{\partial N^R} = \tilde{b}^R(N^L, N^R)$$

.

Hence, we have

$$P^L(N^L, N^R) = c^L + cN^R - N^R \tilde{b}(N^L, N^R)$$

.

Bibliography

[1] Rochet, J.C., Tirole, J. *Platform Competition in Two-Sided Markets*, IDEI Working Papers 152, Institut d'conomie Industrielle (IDEI), Toulouse, 2003.

[2] Weyl, E Glen. *A Price Theory of Multi-sided Platforms*, American Economic Review, 100(4): 1642-72, 2010.