ORIE 6180: Design of Online Marketplaces **Spring 2016**

Lecture 13 — March 16

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13.1 Overview of the last lecture

In the mechanism design section of this course, we studied the single parameter auction. In this case, the VCG auction (the second price auction) maximized social welfare. In terms of revenue, we showed that posted price got within 1/2 of the optimal revenue while the Meyerson auction maximized revenue. However, the Meyerson auction required potential smoothing, finding individual reserves, and calculating virtual valuations which could potentially be very complicated. We also saw how the Meyerson auction could lead to counter-intuitive behavior where the item was not awarded to the person with the highest value.

13.2 Overview of this lecture

In this lecture, we looked at the result in [1] that analyzed a new auction called Buy-in-now or Take-a-chance (BIN-TAC). The setting the authors consider is when there is a small probability a user will have a very high value and a large probability of having a low value. In this case, neither the second price auction with reserve nor the posted price auction have the properties we would like and the Meyerson auction would be quite complicated to run. The main idea is that BIN-TAC can combine the relative benefits of both through price discrimination.

13.3 Setting

Suppose we look at ad-auctions in which the seller has some extra information about the user through user targeting (e.g., using cookies). These auctions are typically quite small with 6-7 bidders. If the seller divulges this information to the bidders then there is a small chance a bidder will have a much higher value for the ad-placement. For example, the user may be located in the same town as the store the bidder is advertising.

13.3.1 General Model

We assume

1. n bidders

2. SPIV - symmetric, private individual valuations
3. Each bidder i has valuation $V_i = XV_H + (1-X)V_L$ where $X \sim \text{Ber}(\alpha)$, $V_H \sim F_H$ with with pdf f_H and support $[\underline{w}_H, \overline{w}_H]$ and $V_L \sim F_L$ with pdf f_L and support $[\underline{w}_L, \overline{w}_L]$. Here, $\overline{w}_L < \underline{w}_H$.

In the more restricted **Two-Type** setting, we assume that $\underline{w}_H = \overline{w}_H = v_H$ and $\underline{w}_L = \overline{w}_L = v_L$. In other words, with probability $1-\alpha$ a bidder has value v_L and with probability α a bidder has value v_H. We consider α to be very small.

13.3.2 Other Auctions

Consider the following auctions under the general model:

- **Second price with reserve (SPA-T)**: this auction will have high social welfare but may often charge a high value bidder a low price and will have low revenue.
- **Posted Price (PP)**: this auction will do better with respect to revenue but could often not sell the item resulting in a *thin market*.
- **Meyerson's Auction**: this auction will maximize revenue but could be very complex to run and may have counter-intuitive behavior.

We want to design an auction that is simple, is robust, has high welfare and revenue, and has proven approximation guarantees with respect to the optimal welfare and revenue. The BIN-TAC auction will use randomization as a tool to price discriminate.

13.4 BIN-TAC

BIN-TAC(p, r, d): In the Buy-it-now (BIN) stage, the seller lists a posted price p. If one bidder enters the auction in this stage, the seller gives the item to that bidder with price p. If two or more bidders enter, then the item is sold using a second price auction with reserve p. If no bidders entered the BIN stage, then the Take-a-chance (TAC) stage happens where the item is rewarded to a random bidder among the highest d bidders with price $\max\{r, b_{d+1}\}$, where b_{d+1} is the $(d+1)$st highest bid.

Note if $d > n$, then the item is sold with probability n/d (to a random bidder among all n). If $d = \infty$, then the TAC stage is not run.

13.4.1 Optimization under the Two-Type Model

Under the Two-Type model, we look at optimizing revenue over p, r, and d. It is always optimal to set $r^* = v_L$. Therefore, we want to solve the maximization problem

$$\pi^* = \max_{d,p} \left[\min\left(\frac{n}{d}, 1\right)(1-\alpha)^n v_L + n\alpha(1-\alpha)^{n-1}p + v_H(1-(1-\alpha)^n - n(1-\alpha)^{n-1}\alpha)\right]$$
$$\text{s.t. } v_H - p \geq \frac{1}{d}(v_H - v_L)$$

Here, the constraint ensures that the auction is IC. Since the objective function is increasing in p, the optimal value of p given d is

$$p^*(d) = v_H - \frac{1}{d}(v_H - v_L).$$

Given $p^*(d)$, the objective function becomes increasing in d if $d \leq n$. Using this, we find

$$d^* = \begin{cases} n & \text{if } \alpha v_H < v_L, \\ \infty & \text{otherwise} \end{cases}.$$

13.5 Auction Comparisons

Theorem 13.1. *BIN-TAC is optimal with respect to revenue under the Two-Type model.*

If $\alpha v_H \geq v_L$, then BIN-TAC has the same revenue as SPA-T since only the first stage is run. Suppose that $\alpha v_H < v_L$ and let q_i be the probability that exactly i bidders have $X_i = 1$. Then, the revenue under BIN-TAC is

$$\pi^*_{BIN-TAC} = (q_0 + q_1/n)v_L + (1 - q_0 - q_1/n)v_H.$$

In contrast, the SPA-T auction has revenue

$$\pi^*_{SP} = (q_0 + q_1)v_L + (1 - q_0 - q_1)v_H < \pi^*_{BIN-TAC}.$$

Suppose instead that the information about the user is not released. This is called *bundling*. Then every bidder has value $V = (1-\alpha)v_L + \alpha v_H$ which will also be the revenue under any reasonable auction. However,

$$q_0 + q_1/n < 1 - \alpha$$

so $\pi^*_{BIN-TAC} > V$ and BIN-TAC has higher revenue.

Bibliography

[1] Celis, L. E., Lewis, G., Mobius, M., and Nazerzadeh, H. Buy-It-Now or Take-a-Chance: Price Discrimination Through Randomized Auctions, Management Science, 60(12): 2927-2948, 2014.